TRINITY COLLEGE

Year 12 Mathematics Applications
Test 6 2017

Calculator Free

Project Networks and Assignment Problems

STUDENT'S NAME Solutions

DATE: Wednesday 30th August **TIME:** 50 minutes **MARKS**: 50

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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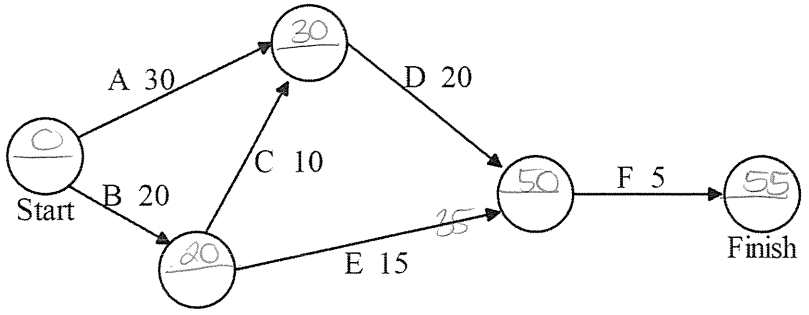
1. (5 marks)

In the following project network the times for activities A – F are given in minutes.



(a) State the minimum completion time and critical path(s) for the project. [2]

ADF & BCDF 55 min ✓

1 mark for both

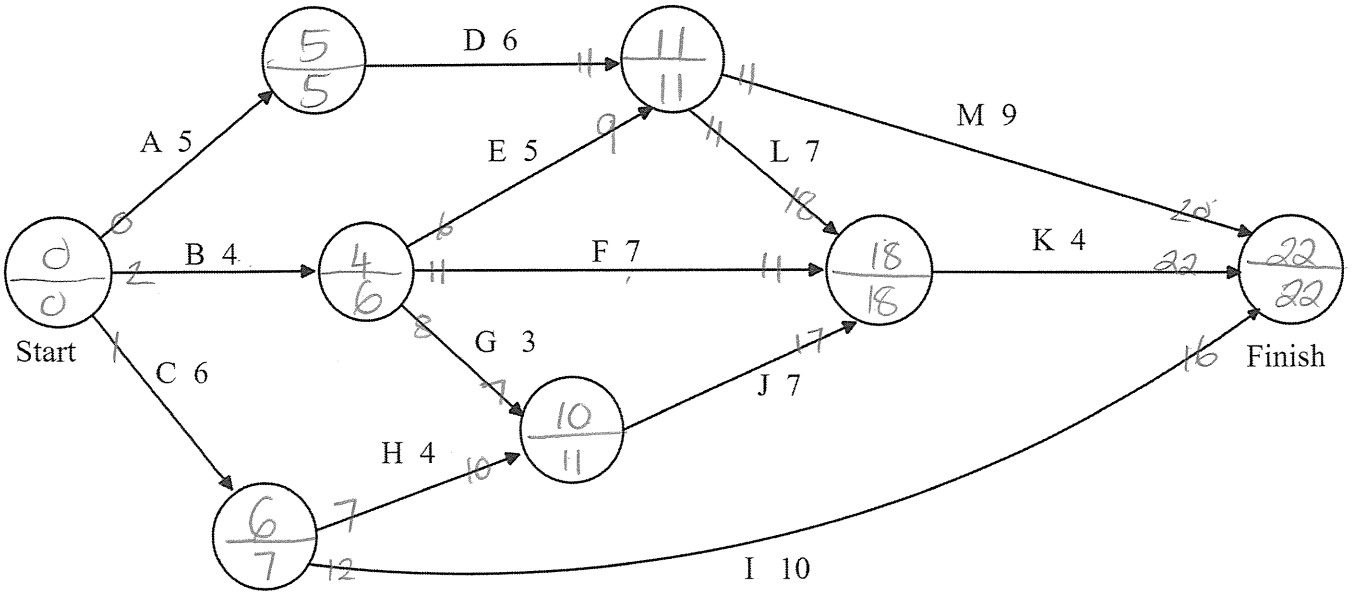
(b) Complete the precedent table below. [3]

Activity	Immediately Preceded by:
A	—
B	—
C	B
D	AC
E	B
F	DE

-1 per error

2. (11 marks)

A project consists of 13 activities, A to M. The project network representing the scheduling of these activities is shown below. The times are in hours.



(a) State the critical path and the minimum completion time for this project. [2]

22 hours ADLK

(b) Determine the:

(i) earliest starting time for activity F. [1]

hour 4

(ii) latest start time for activity I. [1]

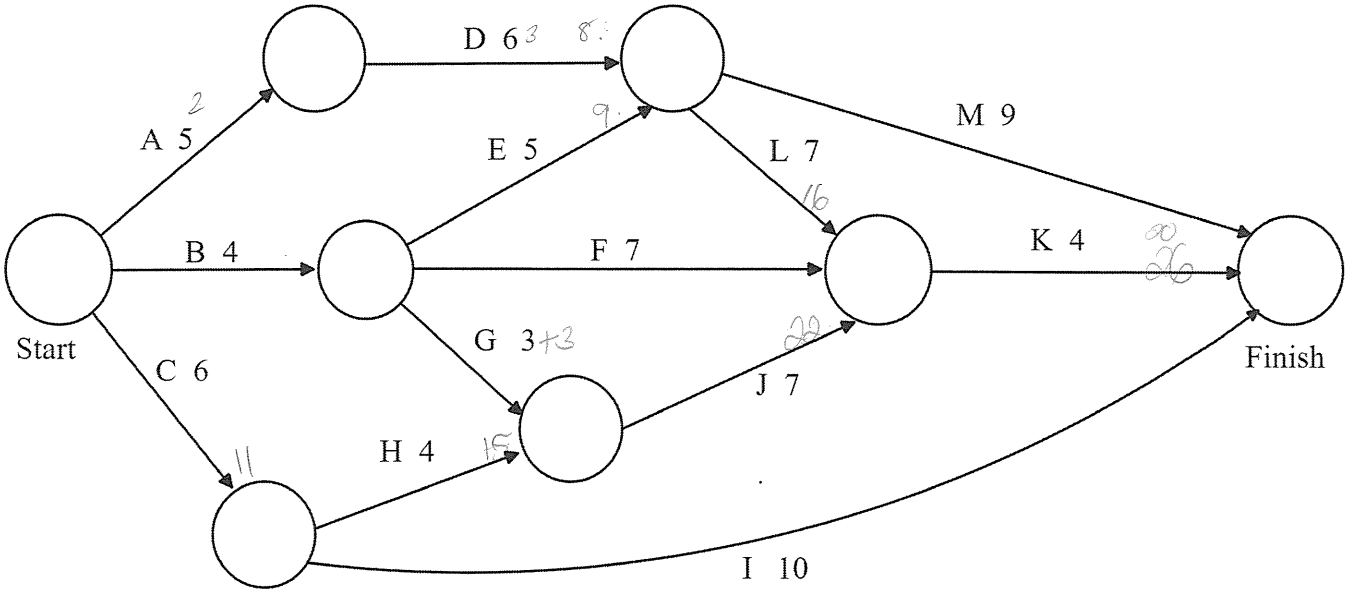
hour 12

(iii) float time for activity H. [1]

1 hour

C+H+J < 18
6+4+7 < 18

A copy of the diagram is given below. It can be used to assist with parts (c) to (e).



(c) If activity C is delayed by 5 hours, what effect will this have on the critical path and the completion time? [2]

✓ - It will add 4 hours, now 26 hours

✓ - New path CHJK

(d) If activity G is delayed by 3 hours, what is the maximum number of hours activity J can be delayed without effecting the minimum completion time? [2]

$B + G + 3 + J < 18$
$4 + 3 + 3 + 7 < 18$
$17 < 18$

∴ J can be delayed by 1 hour ✓✓
without changing the completion time

(e) A volunteer has offered three hours of their time to help reduce the time of one activity by 3 hours. Which task(s) could the organiser assign the volunteer to get the greatest reduction in the minimum completion time? [2]

✓ states 1 task
✓ states all tasks

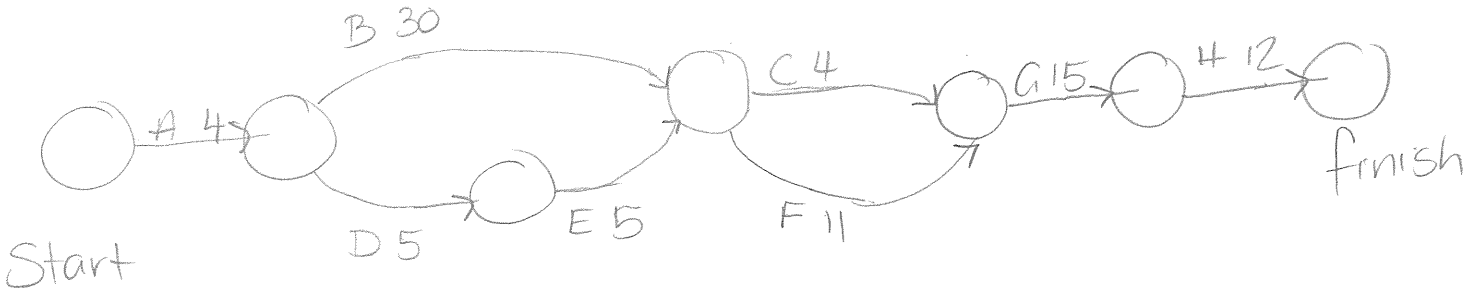
Activity	Reduction (hours)
A	2 hours
D	2
L	2
K	2

∴ Any task on critical path (ADLK) for a maximum reduction of 2 hours

3. (5 marks) Draw circles

The table below describes the various activities involved in assembling a computer. Construct a project network for this table of activities.

Activity	Description of Activity	Immediate Predecessor	Time (min)
A	Install mother-board	-	4
B	Test hard drive	A	30
C	Install hard drive	B,E	4
D	Install I/O ports	A	5
E	Install GPU	D	5
F	Test GPU	B, E	11
G	Install operating system	C,F	15
H	Test assembled computer	G	12



√√√ Arcs correct
√ labels arcs with Activity
√ labels arc with times

4. (8 marks)

A company has four factories, F1, F2, F3 and F4, each of which can produce four products, A, B, C and D. The sales revenue and production cost, in thousands, of the factories differ from one plant to another and is given in the tables below.

	Sales Revenue in Factories			
Product	F1	F2	F3	F4
A	50	68	49	62
B	60	70	51	74
C	55	67	53	70
D	58	65	54	68

	Production Cost in Factories			
Product	F1	F2	F3	F4
A	49	60	45	61
B	55	63	45	49
C	52	62	49	68
D	55	64	48	66

(a) Given that Profit = Revenue – Cost, fill in the missing values in the table below. [2]

	Profit in Factories			
Product	F1	F2	F3	F4
A	1	8	4	1
B	5	7	6	25
C	3	5	4	2
D	3	1	6	2

largest number

(b) By showing use of the Hungarian Algorithm, determine the optimal allocation of products to factories and the maximum profit associated. [6]

① Cost matrix ✓

$$\begin{bmatrix} 24 & 17^* & 21 & 24 \\ 20 & 18 & 19 & 0^* \\ 22 & 20^* & 21 & 23 \\ 22 & 24 & 19^* & 23 \end{bmatrix}$$

② Subtract row

$$\begin{bmatrix} 7 & 0^* & 4 & 7 \\ 20 & 18 & 19 & 0^* \\ *2 & 0 & 1 & 3 \\ 3 & 5 & 0^* & 4 \end{bmatrix} \checkmark$$

③ Subtract Column

$$\begin{bmatrix} 5 & 0 & 4 & 7 \\ 18 & 18 & 19 & 0 \\ 0 & 0 & 1 & 3 \\ 1 & 5 & 0 & 4 \end{bmatrix} \checkmark$$

④ row = lines
∴ Solution found

drawing lines ✓

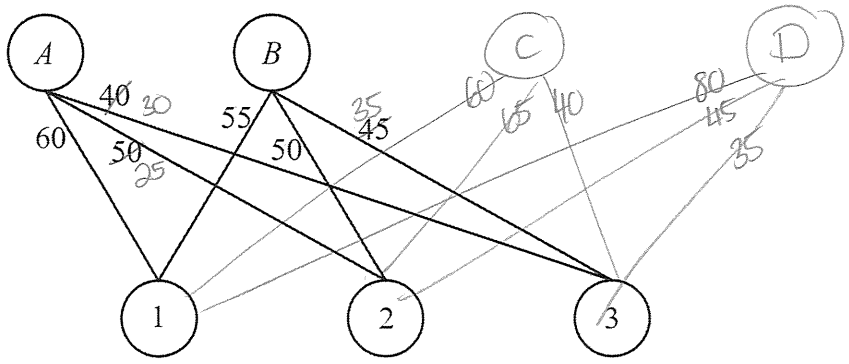
F1 C
F2 A
F3 D
F4 B ✓

3+8+6+25 = 42
∴ $42 000 ✓ profit

-1 if $42

5. (11 marks)

Mr Smith is getting quotes to get some jobs done around his house. He needs the gutters cleaned (1), the windows washed (2) and his backyard weeded (3). He wants it all done on the same day so he will need to hire a different company to complete each job. The quotes (in dollars) for two companies, Aaron's A Team (A) and Bill's Bleachers (B) are shown in the bipartite graph below.



(a) If Mr Smith can complete one of the jobs himself to help minimise the cost, state:

(i) which of the three jobs he should do. [1]

Job 1: Gutters ✓
(most expensive)

(ii) the minimum cost and how he should allocate the remaining two jobs. [2]

A 2
B 3 ✓
$60 ✓

Mr Smith is worried he won't have time to do any of the jobs himself so he decides to get two more companies to quote on the jobs. Their quotes are shown in the table below.

	Carl's Cleaners (C)	Damon's Dynamites (D)
Gutters ①	$60	$80
Windows ②	$65	$45
Weeds ③	$40	$35

(b) Add the new information to the bipartite graph above. [2]

adds C & D ✓
add values ✓

(c) By showing use of the Hungarian Algorithm, determine the optimal allocation of the three jobs to three different companies and the minimum cost associated. [6]

① Cost matrix

	A	B	C	D
1	60	55	60	80
2	25	50	65	45
3	30	35	40	35

② Row ≠ Columns
∴ add dummy row of zeros

③ New Cost Matrix

60	55*	60	80
25*	50	65	45
30*	35	40	35
0	0	0	0

④ Subtract Smallest in row

~~5~~	0	5	~~25~~
0	25	40	20
0	5*	10	5
~~0~~	~~0~~	~~0~~	~~0~~

⑤ Subtract smallest in column (no change) ✓

⑥ lines < row ∴ solution not found

⑦

A	B	C	D
10	0	5	20
0	20	35	15
0	0	5	0
5	0	0	0

✓

⑧ lines = rows
∴ solution found

B 1
A 2
D 3 ✓

25 + 55 + 35
= $ 115 ✓

-1 if allocate C4

-2 if no dummy row
i.e 3x4 matrix

-3 if no dummy row & remove a company
i.e 3x3 matrix

6. (10 marks)

Five jobs (1 to 5) are to be completed on five different machines (A to E). The associated costs are shown in the following table.

Job/Machine	A	B	C	D	E
1	9	(3)	1	13	1
2	(1)	17	13	20	5
3	0	14	8	11	(4)
4	19	3	(0)	5	5
5	12	8	1	(6)	2

The first few steps of the Hungarian Algorithm have been carried out to minimise the cost of completing the jobs. The resulting matrix below can be used to determine if an optimal solution has been found.

(a) Explain why the resulting matrix cannot be used to determine the optimal solution. [2]

$$\begin{bmatrix} 8 & 0 & 0 & 7 & 0 \\ 0 & 14 & 12 & 14 & 4 \\ 0 & 12 & 8 & 6 & 4 \\ 19 & 1 & 0 & 0 & 5 \\ 11 & 5 & 0 & 0 & 1 \end{bmatrix}$$

lines < rows

∴ solution not found.

(b) Make any necessary adjustments needed to produce a matrix that can be used to determine an optimal solution. [4] [3]

$$\begin{bmatrix} 12 & 0 & 0 & 7 & 0 \\ 0 & 10 & 8 & 10 & 0 \\ 0 & 8 & 4 & 2 & 0 \\ 23 & 1 & 0 & 0 & 5 \\ 14 & 5 & 0 & 0 & 1 \end{bmatrix}$$

lines = rows

∴ solution found

(c) State all optimal allocations and the minimum cost. [4] [5]

(1)	(2)	(3)	(4)
A 2	A 3	A 2	A 3
B 1	B 1	B 1	B 1
C 4	C 4	C 5	C 5
D 5	D 5	D 4	D 4
E 3	E 2	E 3	E 2

1+3+0+6+4 = 14 units